DESIGN OF AN INDENTOR VISCOMETER
FOR
DETERMINING THE VISCOSITY OF A FLUID

by

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CHAPTER I

INTRODUCTION

Purpose Of The Study:

Rheology is the science of the deformation and flow of matter. Rheology today is at a turning point. Many of the problems of the past have received satisfactory answers. The general phenomenological theory for time-independent flow of all types in capillary and rotating-cylinder viscometers has been developed.

In spite of the fact that many advancements have been made in the field of rheological measurements; there are very few viscometers commercially available which can determine the viscosity of fluids ranging up to 10^{10} poises which will produce satisfactory results.

In view of this, it was proposed to design a viscometer for determining viscosity up to 10^{10} poises. On the basis of theoretical analysis of Goudy and Kirmser,[14] a viscometer was designed and fabricated at Kansas State University. The author used this viscometer for purpose of determining viscosity of various fluids. The purpose of this report is to present the details of the experiment performed and data obtained thereby. The data was analysed statistically using computer program to obtain the results. In order to compare these results the author used Brookfield Viscometer also. The details of results of both has also been discussed.

Review Of Literature:

What is Viscosity:

Viscosity is the property of a fluid (liquid or gas) that mainly characterizes its flow behaviour and by virtue of which it offers resistance to shear stress. Viscosity defines the internal friction between the molecules of a fluid. Whenever any layer of a fluid is caused to move over another, a frictional resistance is offered by the accompanying layers and tend to be carried along too. Due to this resistance are developed velocity differences within the fluid and forms the basis of the quantitative measure of viscosity.

The measurement of viscosity is of considerable importance in both industrial production and fundamental science. Viscosity helps in determining the forces to be overcome when fluids are used in pipelines, bearings etc. It also helps in controlling the flow of liquid in processes like spraying, injection moulding, extrusion and surface coating. It has an important bearing with mixing and heat transfer characteristics of fluids. Viscosity influences the size, shape and arrangement of the molecules in fluids and as such it is of great importance to a chemist.

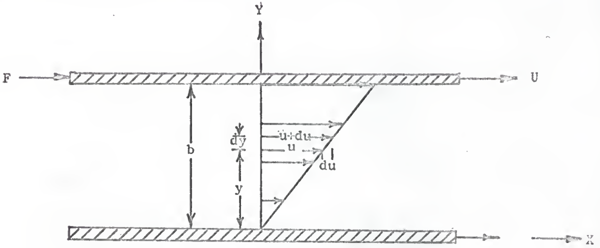
Newton's Law of Viscosity:[1]

It states that for a given rate of angular deformation of fluid the shear stress is directly proportional to the viscosity.

Mathematically

$$\text{Viscosity } (\mu) = \frac{\text{shearing stress}}{\text{rate of shearing strain}}$$

Consider two parallel plates of surface area A filled with a fluid as shown in Fig. 1. A tangenlial force F is applied to the top plate to move it at a constant velocity U parallel to the stationary bottom plate.



Stationary Plate

Fig. 1. Flow between parallel plates

Neglecting gravitational forces

$$\frac{F}{A} \propto \frac{U}{b}$$

or

$$\frac{F}{A} = \mu \frac{U}{b} = \tau$$

Where the constant of proportionality is defined as the coefficient of viscosity (μ).

If the velocity distribution is as shown in Fig. 2 then $u/_b = \frac{du}{dy}$. Therefore

$$\tau = \mu \frac{du}{dy}$$

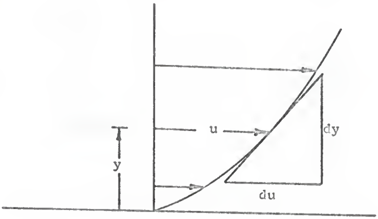


Fig. 2. Velocity distribution

Fluids which obey Newton's Law of viscosity are classified as Newtonian. In Newtonian fluid there is a linear relation between the magnitude of applied shear stress and the resulting rate of deformation[2] as shown in the Fig. 3,

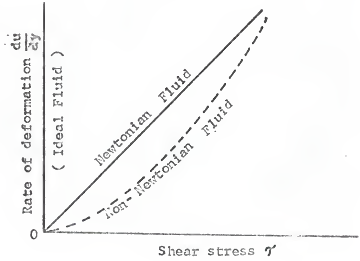


Fig. 3. Rheological diagram

while the non-Newtonian fluid follows a non-linear path. Primarily this discussion is limited to Newtonian fluid only.

Measurement of Viscosity:

Experimently the viscosity of a fluid can be measured with an instrument called viscometer. There are a variety of viscometers available and can be classified into three main types:

1. Capillary Viscometer
2. Rotational Viscometer
3. Falling Ball Viscometer

Discussion of each type in detail, follows.

Capillary Viscometers:

Introduction:

Probably the first scientific experiment in which a capillary or tube was used to measure flow was made by Hagen[3] in 1839, followed shortly then after by Poiseuille's work. Poiseuille studied capillary flow problems in order to understand better the circulation of blood through capillary vessels in the human body. He discovered the relation (known as the Hagen-Poiseulle Law) between flow rate and pressure drop for capillary flow after experimenting fortunately, with water instead of blood. Probably he would not have arrived at this result had he used blood-a non-Newtonian fluid! This discovery was the foundation of capillary viscometery. Following Poiseuille, Wiederman and later Hagenbach derived a theoretical formula for Poiseuille's discovery based on Newton's definition of viscosity. Until 1890, when Couette devised a new method based on a system of two concentric cylinders, capillary flow was the only widely used technique in viscometry.

Capillary viscometers are the most commonly used type for measurements on Newtonian liquids. They are comparatively simple and inexpensive, they

require only a small quantity of test liquid, temperature control is easy and a full mathematical treatment is possible. In general, the liquid is made to flow through a capillary tube under a known pressure difference and the rate of flow is measured, usually by noting the time taken for a given volume of the liquid to pass a graduation mark. In certain types of instrument the liquid is forced through the capillary at a predetermined rate and the pressure drop thereby produced across the capillary is measured. A complete capillary viscometer consists of five essential parts:

1. Fluid reservior
2. Capillary of known dimensions
3. A unit for controlling and measuring the applied pressure
4. A unit for determining flow rate, and
5. A unit for controlling temperature

Commercially available capillary viscometers may be divided into three main types.

a. The cylinder-piston (or plunger) variety
b. Glass capillary units, and
c. Orifice viscometers

a. Flow in a Capillary Tube (Poiseuille's Equation[4])

Consider a cylinderical capillary of radius 'a' and length 'l' with a pressure difference P between the ends. It is assumed: (i) that the flow is every where parallel to the axis of the tube; (ii) that the flow is steady, i.e. there is no acceleration of the liquid at any point; (iii) that there is no slip at the walls, i.e. the liquid in contact with the walls of the capillary is stationary; (iv) that the liquid is Newtonian, i.e. the ratio of the shearing stress F to the rate of shear D is a constant η

the viscosity coefficient.

Let the velocity of the liquid at a distance r from the axis of the capillary be v

Then

$$D = \text{velocity gradient} = -\frac{dv}{dr}$$

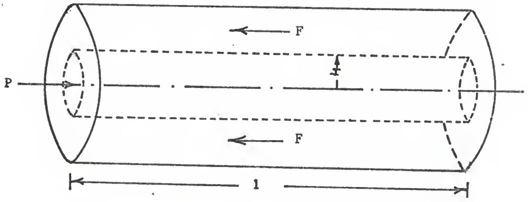


Fig. 4. Flow in a Capillary Tube

Consider the forces acting on a cylinderical element of length 1 and co-axial with the capillary. That due to the applied pressure P is $\pi r^2 P$ acting in the direction of motion. That due to the viscous resistance of the surrounding liquid is F x area = F x $2\pi r l$. Since there is no acceleration of the liquid, these forces must balance, so that

$$F \times 2\pi r l = \pi r^2 P$$

or

$$F = \frac{Pr}{2l}$$

Newtonian behaviour requires that

$$F = \eta D, \quad \text{so that}$$

$$\frac{dv}{dr} = - \frac{Pr}{2\eta l}$$

Thus the shearing stress and rate of shear are directly proportional to r and are zero on the axis of the capillary.

Integrating the above equation, using the condition that at the wall of the capillary,

$$r = a \text{ and } v = 0, \text{ gives}$$

$$v = \frac{P\,(a^2 - r^2)}{4\eta l}$$

The velocity distribution across the capillary is therefore parabolic. The volume of liquid flowing in unit-time between radii r and r + dr is $2\pi rvdr$ and therefore the overall flow (Q $cm^3 sec^{-1}$) through the capillary is given by

$$Q = \int 2\pi rvdr$$

$$= \frac{2\pi P}{4\eta l} \int r(a^2 - r^2)dr$$

$$= \frac{\pi Pa^4}{8\eta l}$$

This is known as Poiseuille's equation.

Sources of Errors:

1. Streamline and turbulent flow: The derivation of Poiseuille's equation assumes that flow in capillary is every where streamline. It is found experimentally that deviation from this law occur at high rates of flow, this was shown by Osborne Reynolds (1883) to be due to a change from

streamline to turbulent flow.

2. Kinetic Energy Correction: Poiseuille's Law applies strictly only to that portion of the tube in which the velocity has become constant. In most types of viscometers, part of the applied pressure is used to give kinetic energy to the liquid, so that a correction must be applied in order to obtain the pressure used in overcoming the viscous resistance.

3. Viscous End Effects: In addition to the work done in overcoming the viscous resistance in the capillary itself, a small amount of energy is expended in overcoming the viscous forces between the converging diverging streamlines at the entrance and exit respectively of the capillary.

   Couette was the first to suggest a correction to account for entrance loss for a Newtonian fluid in a capillary, and it is generally known by his name.

4. Hydrostatic Head Correction: In viscometers involving applied pressure the effective pressure drop across the capillary is the sum of the externally applied pressure and due to the head of liquid in the viscometer, the latter of course normally decreasing during the experiment and for which decrease correction must be made.

5. Drainage Errors: When a liquid is drained from the bulb, some portion of the liquid adheres to its walls. Several types of viscometers include a second bulb of similar shape above the main bulb to make drainage conditions the same when the level of liquid passes the upper & lower marks.

6. Surface-Tension Correction: In viscometers in which there is free discharge of the liquid from the capillary into air, the formation of

droplets is associated with a small pressure drop, and the rate of flow for the same total pressure is slightly decreased by an amount dependent on the surface tension of the liquid. This effect is eliminated in most viscometers by using submerged discharge of the liquid from the capillary.

7. Wall Effect: Analysis of capillary-viscometer data is based on the assumption that the velocity is zero at the wall of the capillary. If a fluid has a finite velocity at the wall, this change is boundary condition should be taken into account in deriving viscometry equation.

b. Glass Capillary Viscometers:

Most glass capillary viscometers are operated by the force of gravity only because of the small driving force, this class of devices is useful for low-viscosity liquids ranging from 0.9 to 16,000 centistokes. The viscometers can often be operated by application of external pressure in addition to the hydrostatic head. In this way, the range of the viscometers can be increased considerably, perhaps as much as 40 times.

The principal of these instruments is derived from the viscometer origionally used by Ostwald[3]. Basically, the viscometer consists of reservoir bulbs and a capillary in a u-tube arrangement, as shown in the Fig. 5. The efflux time of a fixed volume of liquid under an exactly reproducible mean hydrostatic head is measured.

This origional Ostwald Viscometer has been modified in many ways to minimize certain undesirable effects in viscosity measurements, to increase the range of viscosity, to meet the specific requirements of certain test liquids. For instance, Caxnon and Fenske[5] modified the Ostwald Viscometer so that the upper and lower bulbs be on the same vertical axis in order to

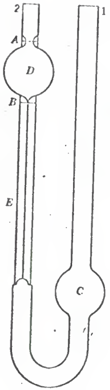


Fig. 5. Ostwald Viscometer[3]

reduce the error in the mean head caused by deviation of the viscometer from the vertical position. For opaque fluids, the viscometer was modified to have reverse flow so that movement of the meniscus can be clearly observed.

The hydrostatic head available for liquid flow in a glass capillary viscometer varies with time. The degree of hydrostatic head variation during the time of efflux in a measurement depends upon the design of the viscometer. For a Newtonian liquid, the viscosity of which is constant, variations in the pressure head have no effect on the measurement, irrespective of the fluid, as long as a constant volume of the test liquid is used. For a non-Newtonian liquid, it differs when going from one liquid to another.

Glass Capillary Viscometers have been used for determining viscosities of Newtonian liquids because of their excellent accuracy, relative cheapness, and simple operation.

Measurement By Gravity:

Operation: Although specific details of operation vary considerably for the viscous types of glass capillary viscometers, the general operating procedure is the same. Fig. 5 shows a schematic diagram of an Ostwald Viscometer.[3] Typical steps in making a measurement are (1) a sample of liquid to be tested is charged through tube 1 to bulb C; (2) the viscometer with the sample is immersed in a constant-temperature bath to attain the desired temperature; (3) the liquid is raised in tube 2 by applying suction until the level is above mark A, and by removing the suction the liquid starts to flow through the capillary E; (4) the efflux time of the volume of bulb D between the A and B marks is measured.

In order to obtain an accurate and reproducible result, the hydrostatic head should be exactly the same for all measurements. This necessitates, for most viscometers, always charging the instrument with a given amount of fluid. The kinematic viscosity[3] of the liquid is calculated from the measured efflux time, using the equation given below.

$$\nu = \eta/\rho = k\theta - K/\theta^m \qquad (\theta \text{ is efflux time })$$

where the value of m=2 for capillaries with trumpet-shaped ends; m=1, for square ends, and in general it is assumed that m=1 in relative viscosity measurements with capillary viscometers.

K is assumed to vary in proportion to the square root of the Reynolds number.

Calibration:

There are two procedures available for calibration.

(1). By Means of Standard Fluids: The viscometer is calibrated by measuring the efflux time of a standard fluid. In order to determine the two coefficients in the equation

$$\nu = \eta/\rho = k\theta - K/\theta^m$$

assuming that m=2, two measurements are made with oils differing by approximately fivefold in viscosity. The coefficient k & K are calculated for m=2 by solving simultaneous equations obtained by subsituting viscosities and efflux times of the reference materials into the above equation.

(2). "Step-up" Procedure:[3,6] In cases in which the kinetic energy correction is negligible for a set of viscometers (e.g., a large length-to-diameter ratio for the capillary), the calibration is done

successively, starting with water as the reference fluid in the viscometer with the smallest-diameter capillary. First, the efflux time of freshly distilled water is determined with smallest-capillary.

Assuming a negligible kinetic-energy correction, the constant k is determined in the above equation from the measured time with water. Using this value of k, the viscosity of a more viscous liquid is determined. This viscosity value is then used in calibrating a second viscometer with larger capillary diameter. After the second has been calibrated, another kinematic value can be determined for a previously unstandardized oil of still higher viscosity. This value is than used to calibrate the third viscometer and so on. Steps between successive calibration constants or viscosities increase by a factor of three or less until the desired viscosity range is covered.

Orifice Viscometers[3]:

In the course of development, of certain process industries, the problems of measuring and controlling flow properties of various classes of fluids have been encountered. This has led to the development of instrument for resolving these problems, and a number of them have come into general use. For instance, the oil industry adopted the method of measuring viscosity with short-tube or orifice viscometer. These instruments are now common in industry: e.g., the Saybolt-universal and Furol in the United States;Redwood No. 1 and No. 2 in Great Britain, Engler in Germany, and Barbey in France. In the paint industry cup-type viscometers have been developed and are widely employed at the present time.

The origional design concepts of those viscometers were derived from the Hagen-Poiseuille Law which states that the efflux time of a fixed volume through a capillary is proportional to the viscosity of the fluid. Unfortunately, the viscometers that were developed consisted of short capillaries or orifices. Flow in these viscometers does not obey the Hagen-Poiseuille Law and efflux times are not in any simple relation to the viscosities. An added factor in the development of such viscometers was the practical requirement that the method should be simple, quick and reliable.

In these viscometers the orifice length does not exceed 10 times the diameter of the orifice. The hydrostatic head, i.e., the driving force causing fluid to flow, is consumed to an appreciable extent at the orifice enterence. The friction loss due to this effect is a function of the cross-sectional area ratio of cup-to-orifice, velocity of the fluid, shape of the orifice-enterence port of the viscometer, and properties of the fluid. Furthermore, the situation becomes more complicated because of the varying hydrostatic head during an experimental run. For this reason, the efflux time readings for these viscometers are not in the same relation to the driving force as with the usual capillary viscometers, and conversion formulas or tables must be consulted in comparing results.

This type of viscometer, however, is widely used because of simplicity and inexpensive operation. These viscometers are mostly used for Newtonian fluids.

Principle of Operation:

Viscometers of this type consists of a reservoir and orifice (about 10 diameters long at most) with or without a temperature-control jacket

and a receiving flask. This method of operation is nearly the same for all.

(1). The liquid under test is poured into a cup sorrounded by a water or oil bath providing temperature control.

(2). The level of the liquid in the cup is adjusted to a definite height.

(3). When the desired temperature is attained, the valve at the base of the cup is opened.

(4). The time required for a specified volume of liquid to discharge through orifice into a measuring vess el placed below is measured.

The measured efflux time, generally in seconds, is a purely arbitrary expression of the viscosity. In order to convert these figures into absolute units, an empirical formula for each instrument is derived. The equation is usually in the form of

$$\nu = \eta/\rho = k\theta - K/\theta$$

This form is the same as that for glass capillary viscometers with the difference that an appreciable end effect is reflected in k. So it requires calibration for various ranges of efflux time. For approximate values viscosity conversion charts are available for reference for orifice viscometers.

Rotational Viscometers:

Sir Isaac Newton first observed the relation between velocity and resistance to flow for a rotating solid cylinder in a uniform and infinite fluid. He showed the exponential decrease in rate of shear for the rotating concentric cylindrical laminar shells of fluid as one moves away from the cylinder and into the fluid. In fact a complete solution to Margule's

equation was not arrived at and it was Stoke's[7] who discovered it later.

Nearly two hundred years elapsed before the first practical rotational viscometer was devised by Couette[3] in 1890. Couette's concentric-cylinder viscometer consisted of a rotating cup and an inner cylinder which was supported by a torsion wire and rested in a point bearing in the bottom of the cup. In this unit the principle of guard rings was utilized to eliminate end effects as shown in the Fig. 6. The guard rings F and F' are fixed and do not move, where as the inner cylinder, represented by A, turns in proportion to the viscosity of the fluid in the gap until the opposing forces of the torsion wire come in balance and there is no net movement of the inner cylinder. When the system is thus balanced the fluid in the gap flows by the stationary inner cylinder and the guard rings. There is no flow over the top and bottom of the inner cylinder since the fluid inside the guard rings is trapped,and is stationary. Couette's design enabled him to calculate the apparent viscosities of non-Newtonian samples with only a small error because of the very small ratio of gap to inner radius.

In 1913 Hatscheck[8] described a modified version of the Couette viscometer which improved on the design of the guard rings and minimized effects with very low viscosity fluids. Until 1940 there was not much literature on rotational viscometery. Since then a good many designs have appeared, some of which are available commercially.

Theory:

A rotating body, immersed in a liquid, experiences a viscous drag or retarding force. The amount of viscous drag is a function of the speed of rotation of the body.

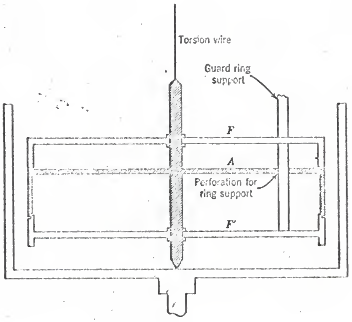


Fig. 6. Couette's Viscometer[3]

Rotational viscometers[4] are usually more elaborate than the capillary type. When used with Newtonian fluids they are often less accurate, but they have a number of advantages including: (1) measurements are made under steady state conditions; (2) during a given measurement variation in shearing stress throughout the material may be made quite small; (3) the same sample may be used for measurement at different shear rates or for continuous measurements on materials whose properties change with time; (4) measurements made on non-Newtonian materials can be interpreted in absolute units. For these reasons rotational viscometers have become the single most widely used class of instruments for rheological determinations.

Derivation of Basic Equation:

Although rotational viscometry includes spheres, disks, cones and odd-shaped rotors and cups, the most common type is the coaxial-cylinder viscometer. In its simplest form as shown in the Fig. 7, the coaxial cylinder viscometer comprises an inner cylinder (radius R_1cm, height h cm) and an outer cylinder (radius R_2cm). The outer cylinder, which acts as the container for the liquid under test, is rotated at a constant speed (Ωrad sec^{-1}), and the resultant torque (T dyn cm) is measured by the angular deflection of the inner cylinder which is suspended by a fine wire.

The following assumptions are made in order to arrive at the fundamental equation:[4,9] (1) the liquid is incompressible; (2) the motion of the liquid is laminar; (3) the streamline of flow are circles on the horizontal plane perpendicular to the axis of rotation (i.e., the velocity is a function only of radius;radial and axial flows are assumed to be equal to zero); (4) the motion is steady-all time derivatives in the equations of continuity and motion are zero; (5) there is no relative motion between

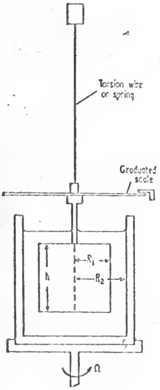


Fig. 7. Co-axial Cylinder Viscometer

the surface of the cylinders and the fluid in immediate contact with cylinders, i.e., no slippage; (6) the motion is two-dimensional (neglecting the edge and end effects).

Let w be the angular velocity of the liquid at a distance r from the axis of rotation, and let the shearing stress and rate of shear at this radius be Fr and Dr respectively. Considering the part of the liquid between the inner cylinder and an arbitrary radius r. The torque exerted on this liquid by the outer layers of liquid is

$$\text{shearing stress x surface area x radius} = F_r \times 2\pi rh \times r = 2\pi r^2 hF_r$$

The torque acting on the inner surface is the measured torque T. Since the motion is assumed steady, these torques must be equal therefore

$$T = 2\pi r^2 hF_r$$

$$F_r = \frac{T}{2\pi r^2 h}$$

Thus the shearing stress is inversely proportional to the square of the distance from the axis of rotation. Now by making the annular gap small as compared to the cylinder radii, it is observed that the shearing stress may be made almost constant throughout the material being sheared. This is in contrast to the capillary viscometer for which the shearing stress varies from zero at the axis to a maximum at the wall of the capillary.

To find an expression for the shear rate, differentiate the linear

velocity with respect to r.

So

$$v = rw$$

$$\frac{dv}{dr} = r\left(\frac{dw}{dr}\right) + w$$

The term $r\left(\frac{dw}{dr}\right)$ represents the shear rate since the other term, w, is the radial velocity gradient of a rigid rotating object.

Thus

$$D_r = r\frac{dw}{dr}$$

For a Newtonian liquid

$$F_r = D_r\eta \qquad \text{and therefore}$$

$$\frac{T}{2\pi r^2 h} = r\frac{dw}{dr}\eta$$

$$\frac{dw}{dr} = \frac{T}{2\pi\eta h r^3}$$

Integrating this using the no-slip boundary condition that

$$w = 0 \quad \text{when} \quad r = R_1$$

$$\text{and} \quad w = \Omega \quad \text{when} \quad r = R_2 \qquad \text{find that}$$

$$T = \frac{4\pi R_1^2 R_2^2 h\eta\Omega}{R_2^2 - R_1^2} = C\eta\Omega$$

Where C is an instrument Constant.

The viscosity is therefore directly proportional to the ratio of torque to angular velocity. It may be noted that the above equation is equally applicable when the inner cylinder is caused to rotate at angular

velocity Ω and the outer cylinder is at rest.

End Effects:

All of the preceeding mathematical derivations assumed characteristics of an infinitely long cylinder in that no account was taken of the drag on the ends of the inner cylinder. Where the term h was used for the height, the expression $h + h_o$ should have been used where h_o is the additional height to be added due to ends. This effect is taken into account in the single instrument constant C which is determined by calibration with one or more liquids of known viscosity.

Temperature Effects:

To a greater or lesser extent, the viscosity coefficient of all fluids are dependent on temperature. So thermostatic control of the material during measurement is highly desirable. While this presents no great difficulty with constant-torque type viscometers in which the cup is stationary, it is less easy with rotating cup instrument.

Friction:

Viscosity measurements involving small torque values are often difficult because of appreciable friction arising in the bearings. While jewelled bearings have been used successfully, one of the most satisfactory method is that described by Oldroyd, Strawhridge and Toms (1951)[4] in which the main spindle carrying the bob is positioned by a number of very small jets of air directed radially on to it.

Wall Effects:

While deriving the general equation it was assumed that the bulk viscous properties of the material apply right up to the surface and that

no slippage occurs at any solid boundary. Wall effects may be suspected if the (F,D) curves determined with different annular gaps differ appreciably.

Time-Dependent Effects (Thixotropy):

There are reversible and irreversible effects varying with time. Of these reversible work softening or thixotropy is most frequently discussed in the literature.

The rotational viscometer is a particularly suitable instrument for the study of thixotropic materials since it permits measurements to be made continuously on the same sample of material under varying shear conditions. There are at least two approaches to the semiquantitative determination of thixotropic changes. These are (1) the measurement of the hysteresis between "up" and "down curves," according to the method of Green[10] and (2) determination of the decay of shear stress as function of time at one or more constant shearing rates (Green and Weltman).[10]

Conclusion:

Rotational viscometers are versatile laboratory instruments for measuring flow properties of many fluids. The simplicity of design and ease of manipulation of the instrument has great appeal to most reheologists. When treating data obtained in the rotational viscometer, it is important to keep in mind the effect of deviations from Newtonian behaviour. The rate of shear is not uniform across the gap and the Newtonian expression cannot be applied directly. Shear stress and the rate of shear should be expressed at the same point in the viscometer and the corrections discussed earlier should be taken into account, the most important of which is the relation of gap size to radius of inner cylinder.

Calibration of the instrument should be done carefully and other factors like temperature turbulence and end effects should also be considered.

Falling Ball Viscometers:

The flow behaviour of a fluid in a rheological instrument is obtained in principle by solving the equation of motion for the given geometry with appropriate boundary conditions. When a solid body is allowed to fall under gravity through a viscous medium a period of initial acceleration is followed by motion at uniform terminal velocity for which the gravitational force is balanced by the viscous resistance. Measurement of this terminal velocity affords a means of determining the viscosity of the medium. Bodies of various shapes may be used; the simplest, both from theoretical and experimental points of view, is the sphere. This method however is limited for use to Newtonian fluids.

Derivation of the Basic Equation:

The principle of the falling-sphere viscometer is the well-known Stoke's Law[11] which relates the viscosity of a Newtonian fluid to the velocity of the falling sphere. According to Stoke's the viscous resistance to the motion of a sphere moving with velocity v is

$$6\pi r\eta v$$

Where r is the radius of the sphere. The driving force, due to the difference in density between sphere and the fluid, is

$$\frac{4}{3}\pi r^3(\sigma - \rho)g$$

Where

σ = density of sphere

ρ = density of fluid

g = acceleration due to gravity

Equating the two expressions, the viscosity

$$\eta = \frac{2gr^2(\sigma - \rho)}{v}$$

Which can be known by measuring velocity (v), r, σ and ρ being fixed.

In the derivation of the above equation it was assumed that the sphere moves with a very slow velocity in the fluid of infinite extent. Since this ideal situation is not realised in actual viscometers, the following corrections should be taken into consideration in calculating the absolute viscosity.

(1). The motion of the sphere relative to the liquid is slow (Finite Reynolds number):

It was assumed in Stoke's Law that the Reynolds number given by

$N_{Re} = 2\ rv\rho/\eta$ is smaller than unity.

To overcome this restriction to a rate of flow Oseen[3] and Goldstein[3] developed a correction factor in terms of infinite series, as given below:

$$\eta_{abs} = \eta / [\ 1 + (3/16)N_{Re} - (19/1280)N_{Re}^2 + (71/20480)N_{Re}^3 \text{ ---------- }]$$

(2). Wall Effects:

It is obvious that in any experimental determination of viscosity the liquid cannot be of infinite extent. For a sphere falling in a cylinder of finite dimensions, the force-resisting motion of the sphere is found to be increased by the drag exerted by the

cylinder wall. Ladenburg[3] showed that the Stoke's viscosity should be corrected as follows:

$$\eta_{abs} = \eta/[1+2.1r/R]$$

Where R is the radius of the container.

(3). Homogenity:

It was assumed that the liquid should be homogeneous. But many liquids change their viscosity with age and others show an apparent viscosity which varies with rate of shear.

(4). End Effects:

According to Ladenburg, the resisting force is increased by the factor as given:

$$\eta_{abs} = \eta/[1+3.3r/h]$$

Where h is the height of the sphere from the bottom of the cylinder.

Falling-ball measurements are generally employed for reasonably viscous materials because of the difficulty of handling the very small balls or of correctly measuring the small differences in density between ball and liquid needed to get a suitably slow rate of flow in a highly fluid medium. This technique is also confined to cases in which disturbance of the fluid prior to the viscosity measurement is to be avoided that rule out the use of other types of viscometers.

CHAPTER II

EXPERIMENTAL PROGRAM

General:

In fact before starting the actual experiment with the Indentor Viscometer, the viscosities of the three Test Fluids were determined with the Brookfield Synchro-Lectric Viscometer.[12] A brief description of the same is given below:

1. Brookfield Synchro-Lectric Viscometer Method:

This method is comprised of the following steps:

(a). Test Samples:

These sample fluids were obtained from Brookfield Engineering Laboratories Incorporated, Stoughton, Masschusettes, U.S.A.

Table No: 1

Test Fluids:

Sr No	Sample	Lot No.	Standard Viscosity	Temperature
1	R_2	120567	29,100 cps	$77^{o}F$
2	H_1	080767	59,400 cps	$77^{o}F$
3	H_2	121367	95,000 cps	$77^{o}F$

(b). Experimental Set Up:

This consisted of the following:

1. Brookfield Viscometer (RV type)
2. Test Fluids

3. Three 600 c.c. beakers

4. A constant temperature bath with thermometer

5. Brookfield Factor Finder

The various working components of the Brookfield Synchro-Lectric viscometer are as shown in the Fig. 8.[12]

(c). Experimental Procedure:

1. The spindle RV-7 (which only covered the range of viscosity to be measured) was attached to the lower shaft while the shaft was held firmly and avoided the side thrusts in order to protect the alignment.
2. The spindle was then lowered into the test-fluid avoiding air bubbles until the fluid level was at the groove cut in the spindle shaft.
3. The viscometer was then levelled with the help of the air bubble level and set the speed of rotation.
4. Then the clutch was depressed and started the viscometer motor. On releasing the clutch the dial rotated till the pointer stabilized at a fixed position on the dial. Different readings were taken for different speeds of rotation ranging from 10-100r p.m. depending upon the test fluid used. At high speed the pointer was held on to the dial when in view by depressing the clutch. Then the motor was switched off and the reading noted when the rotation stopped. The readings were checked by repeating the procedure.
5. The viscosity of the test material was then found by multiplying the dial reading and the multipication factor found by consulting data provided by the Brookfield Company.

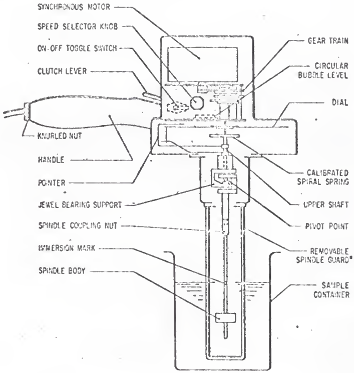


Fig. 8.

SCHEMATIC DRAWING OF THE

BROOKFIELD SYNCHRO-LECTRIC VISCOMETER

(d). Data and Calculations:

Table-2

Observations for Fluid Sample-1 Using Spindle RV-7

Sr No	R.PM	Multipication Factor	Dial Reading	Calculated Viscosity	Standard Viscosity	%age Deviation	Remarks
1	10	4000	7.0	28,000 cps	29,100 cps	3.80	Temp. of bath 77°F
2	20	2000	14.1	28,000 cps	29,100 cps	3.44	
3	50	800	35.4	28,320 cps	29,100 cps	2.68	
4	100	400	70.9	28,360 cps	29,100 cps	2.55	

Table-3

Observations for Fluid Sample-2 Using Spindle RV-7

Sr No	R.PM	Multipication Factor	Dial Reading	Calculated Viscosity	Standard Viscosity	%age Deviation	Remarks
1	10	4000	14.3	57,200 cps	59,400 cps	3.7	Bath Temp. 77°F
2	20	2000	28.7	57,400 cps	59,400 cps	3.37	
3	50	800	71.8	57,440 cps	59,400 cps	3.3	
4	100	400	"	"	"	"	out of Range

Table-4

Observations for Fluid Sample-3 Using Spindle RV-7

Sr No	R.PM	Multipication Factor	Dial Reading	Calculated Viscosity	Standard Viscosity	%age Deviation	Remarks
1	10	4000	22.8	91,200 cps	95,000 cps	4.00	*Bath Temp. 77^{o}F
2	20	2000	45.7	91,400 cps	95,000 cps	3.79	
3	50	800	"	"	"	"	Out of Range
4	100	400	"	"	"	"	

2. Indentor Viscometer Method:

This can be divided into following steps:

(a). General Arrangement:

In this method the same three Test Fluid Samples were used. Besides this there were three indentor sizes, three different weights and three depths of indentation for computation of velocities at these different depths as shown in table-5.

Table-5

Test Fluids and Indentor Sizes

No:	Test Fluid Viscosity (F)	Indentor Radius (R)	Weight of Indentor (W)	Depth of Indentation (D)
1	29,100 cps	0.75 Inches	103.5gms	0.05Inches
2	59,400 cps	1.00 Inches	138.5gms	0.08Inches
3	95,000 cps	1.25 Inches	173.5gms	0.10Inches

Each experimental sequence constituted a combination of one indentor size, one weight of indentor, one depth of indentation and the testing fluid at a time resulting into 81 total number of experiments to be performed. The various combinations of these sequences are shown in table-6.

If the experimental sequence has a combination as

F R W D
1 2 3 2

it means that the fluid used is no:1 (29,100 cps viscosity), Indentor no:2 (1.0"Radius), weight of indentor is no:3 (173.5 gms) and depth of

Table-6

Experimental Sequences:

Seq No:	F	R	W	D	Seq No:	F	R	W	D	Seq No:	F	R	W	D	Seq No:	F	R	W	D	Seq No:	F	R	W	D	Seq No:	F	R	W	D	Seq No:	F	R	W	D	Seq No:	F	R	W	D	Seq No:	F	R	W	D
1	1	1	1	1	10	1	2	1	1	19	1	1	3	1	28	3	2	1	1	37	2	2	3	1	46	3	2	2	1	55	1	2	2	1	64	1	3	3	1	73	3	1	2	1
2	1	1	1	2	11	1	2	1	2	20	1	1	3	2	29	3	2	1	2	38	2	2	3	2	47	3	2	2	2	56	1	2	2	2	65	1	3	3	2	74	3	1	2	2
3	1	1	1	3	12	1	2	1	3	21	1	1	3	3	30	3	2	1	3	39	2	2	3	3	48	3	2	2	3	57	1	2	2	3	66	1	3	3	3	75	3	1	2	3
4	2	1	1	1	13	1	3	1	1	22	2	2	1	1	31	3	3	1	1	40	2	3	2	1	49	3	3	2	1	58	1	2	3	1	67	2	1	2	1	76	3	1	3	1
5	2	1	1	2	14	1	3	1	2	23	2	2	1	2	32	3	3	1	2	41	2	3	2	2	50	3	3	2	2	59	1	2	3	2	68	2	1	2	2	77	3	1	3	2
6	2	1	1	3	15	1	3	1	3	24	2	2	1	3	33	3	3	1	3	42	2	3	2	3	51	3	3	2	3	60	1	2	3	3	69	2	1	2	3	78	3	1	3	3
7	3	1	1	1	16	1	1	2	1	25	2	3	1	1	34	2	2	2	1	43	2	3	3	1	52	3	3	3	1	61	1	3	2	1	70	2	1	3	1	79	3	2	3	1
8	3	1	1	2	17	1	1	2	2	26	2	3	1	2	35	2	2	2	2	44	2	3	3	2	53	3	3	3	2	62	1	3	2	2	71	2	1	3	2	80	3	2	3	2
9	3	1	1	3	18	1	1	2	3	27	2	3	1	3	36	2	2	2	3	45	2	3	3	3	54	3	3	3	3	63	1	3	2	3	72	2	1	3	3	81	3	2	3	3

Note: In the above table the symbols F, R, W and D represent fluid viscosity, Radius of indentor, weight of Indentor and Depth of Indentation respectively and their values have already been tabulated in table-5.

indentation is no:2 (0.08").

The experiments were performed in a purely random manner. This was achieved by consulting the "Random Digits" tables from the book The Compleat Strategyst by J.D. Williams.[13] This random order is as shown in table-7.

Table-7

Random Experimental Order

Exp No	Seq No	Exp No	Seq No	Exp No	Seq No	Exp No	Seq No	Exp No	Seq No	Exp No	Seq No	Exp No	Seq No	Exp No	Seq No	Exp No	Seq No
1	11	10	44	19	56	28	67	37	45	46	09	55	14	64	06	73	41
2	16	11	36	20	31	29	75	38	27	47	15	56	38	65	35	74	70
3	43	12	79	21	28	30	60	39	54	48	24	57	19	66	25	75	07
4	63	13	22	22	72	31	71	40	61	49	66	58	23	67	58	76	30
5	18	14	62	23	64	32	32	41	57	50	34	59	03	68	53	77	02
6	21	15	73	24	20	33	55	42	04	51	42	60	05	69	76	78	69
7	59	16	40	25	37	34	52	43	78	52	29	61	46	70	74	79	80
8	17	17	47	26	51	35	65	44	13	53	50	62	08	71	33	80	77
9	10	18	49	27	48	36	26	45	68	54	12	63	01	72	39	81	81

b. Experimental Set Up:

The appratus used in this method is shown in the Fig. 9 and is listed as under:

1. Indentor Viscometer Unit: This consisted of the following (Fig. 10)
   a. Supporting Rack with levelling base.
   b. An Electromagnet.
   c. Displacement transducer.
   d. Guides for the indentor.
2. Strip chart recorder
   Brush Recorder Mark 1
3. Four $1^1/_2$ volts batteries for supply to the displacement transducer.
4. Supply unit for the Electromagnet NJE corporation Model RB 36-2 0-36 VDC 0-2 Amp.
5. Three glass containers of equal size 90 mm x 50 mm and their covers.
6. Spatula.
7. Kleenex box.

c. Experimental Procedure:

1. The instrument was levelled in order to avoid any error due to misalignment.
2. According to the experimental sequence the required fluid, indentor and weight were selected.
3. After mounting the indentor and the weight, the instrument was calibrated by adjusting the travel 0.1" from the one end of the chart on the recorder to the other end covering 80.0 mm. This calibration was checked once again before starting the experimentation.

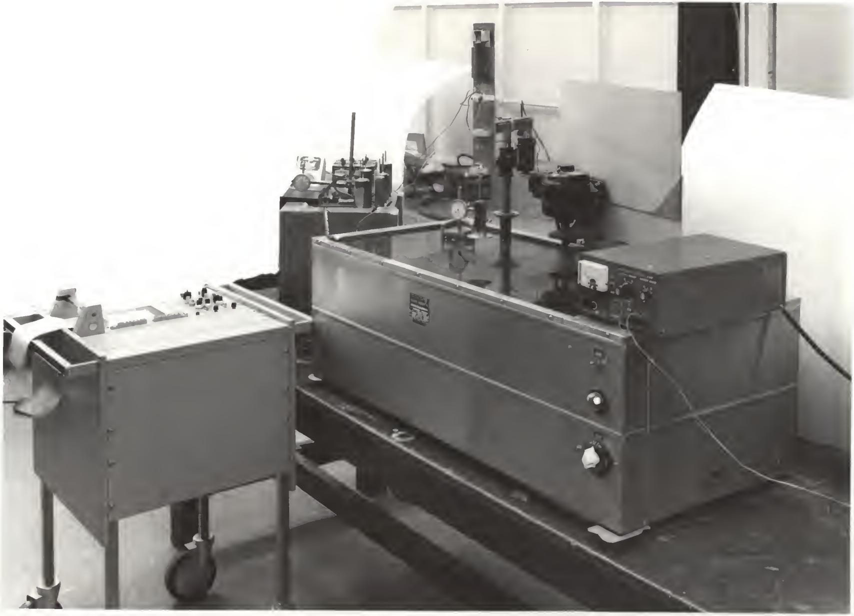


Fig. 9 Indentor Viscometer Set Up

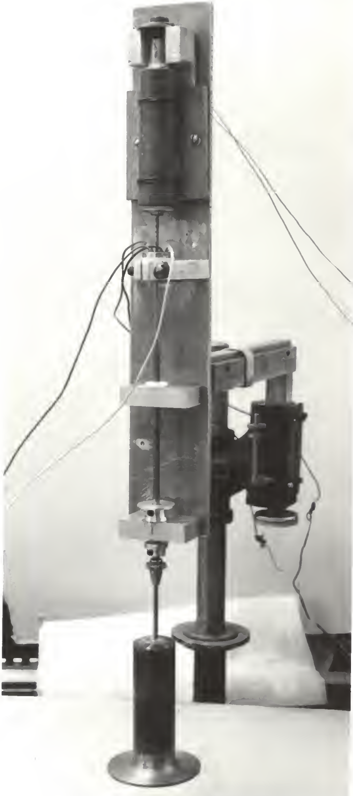


Fig. 10 Indentor Unit

4. The indentor was then adjusted to just touch the fluid surface.

5. The chart speed was set on 200 mm/sec and the indentor released from the magnet by pressing the switch.

6. The velocity profile was observed on the chart and the recorder switched off on completion of the travel.

In this manner all the 81 experiments were performed under similar conditions of temperature and calibrating the instrument every time in order to avoid any variations. Moreover the travel of the indentor disk was kept 0.1" throughout but measurements were made according to the required depths from the chart.

d. Data and calculations are aontained in Appendix B.

CHAPTER III

ANALYSIS

Method Of Least Squares:

The analysis of the data is based upon the derivation of the Indentation of a Newtonian Fluid by a Right circular cylinder.[14]

$$\mu_j = \frac{F_j}{8\, a_j\, v_j} \quad \text{where } j \text{ is the experiment number}$$

μ = Coefficient of viscosity of the fluid

F = Force on the indentor

a = Radius of the indentor

v = Velocity of indentation

It is assumed that viscosity may be dependent upon the fluid used, v, a, F, v^2, a^2, F^2, av, aF, vF and some experimental error e.

Then

$$\mu_j = C_1 L_{1j} + C_2 L_{2j} + C_3 L_{3j} + C_4 a_j + C_5 F_j + C_6 v_j + C_7 v_j^2 + C_8 a_j^2 + C_9 F_j^2 + C_{10} a_j F_j + C_{11} a_j v_j + C_{12} v_j F_j + e_j$$

$$\{\mu_j\} = (L_{1j}\ L_{2j}\ L_{3j}\ a_j\ F_j\ v_j\ v_j^2\ a_j^2\ F_j^2\ a_j F_j\ a_j v_j\ v_j F_j) \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ \cdot \\ \cdot \\ \cdot \\ C_{12} \end{Bmatrix} + \{e_j\}$$

This represents a linear model[15] and in the matrix form it becomes

$$\{\mu_j\} = (\ A\ ) \begin{Bmatrix} C_1 \\ C_2 \\ \cdot \\ \cdot \\ C_{12} \end{Bmatrix} + \begin{Bmatrix} e_1 \\ e_2 \\ \cdot \\ \cdot \\ e_{81} \end{Bmatrix}$$

$$\mu = (\ A\ )\ \{C\} + \{e\}$$

where

μ is the dependent response variables
A is the input independent variables

If errors are present in the dependent variables and independent variables then the problem becomes

$$\{\mu + \delta\} = (\ A + \Delta)\ \{C\} + \{e\} \text{ ------} \qquad (1)$$

in which $\{\delta\}$ is a (81x1) column matrix of unknown errors in μ,

(Δ) is a (81x12) rectangular matrix of unknown errors in (A).

Multiplying (1) by $(\ \tilde{A} + \tilde{\Delta})$, yields

$$(\tilde{A} + \tilde{\Delta})\ \{\mu+\delta\} = (\tilde{A} + \tilde{\Delta})\ (A + \Delta)\ \{C\} + (\tilde{A} + \tilde{\Delta})\ \{e\} \text{ ----} \qquad (2)$$

The estimate $\{\hat{C}\}$ which solves the equation (2) in the least square sense is that for which

$$(\tilde{A} + \tilde{\Delta})\ \{e\} = 0 \quad ,$$

substituting this in (2) and solving

$$\{\hat{C}\} = [(\tilde{A} + \tilde{\Delta})\ (A + \Delta)]^{-1}\ (\tilde{A} + \tilde{\Delta})\ \{\mu+\delta\}$$

$$\{\hat{C}\} = (\tilde{A}\ A+\tilde{\Delta}A+\tilde{A}\Delta+\tilde{\Delta}\Delta)^{-1}\ (\tilde{A} + \tilde{\Delta})\ \{\mu+\delta\}$$

saving the first order terms

$$\{\hat{C}\} = (\tilde{A}\ A+\tilde{\Delta}A+\tilde{A}\Delta)^{-1}\ (\tilde{A} + \tilde{\Delta})\ \{\mu+\delta\} \text{ --------} \qquad (3)$$

Now let

$$(\tilde{A}\ A+\tilde{\Delta}A+\tilde{A}\Delta)^{-1} = (M+\varepsilon)^{-1} \quad \text{where } M = \tilde{A}\ A,\ \varepsilon = \tilde{\Delta}A+\tilde{A}\Delta$$

$$(\tilde{A}A+\tilde{\Delta}A+\tilde{A}\Delta)^{-1} = (M\ (I+M^{-1}\varepsilon))^{-1}$$

$$= (I+M^{-1}\varepsilon)^{-1}M^{-1}$$

By binomial expension and saving the first order terms this becomes

$(\tilde{A}A+\tilde{\Delta}A+\tilde{A}\Delta)^{-1} \simeq (I-M^{-1}\varepsilon)\, M^{-1}$

Therefore (3) becomes

$\{\hat{\hat{C}}\} = (I-(\tilde{A}\ A)^{-1}(\tilde{\Delta}A+\tilde{A}\Delta))\ (\tilde{A}\ A)^{-1}(\tilde{A}+\tilde{\Delta})\ \{\mu+\delta\}$

On multiplication and leaving the terms other than the first order, yields

$$\{\hat{\hat{C}}\} \simeq [(\tilde{A}\ A)^{-1}\tilde{A} + (\tilde{A}\ A)^{-1}\tilde{\Delta} - (\tilde{A}\ A)^{-1}(\tilde{\Delta}A+\tilde{A}\Delta)(\tilde{A}\ A)^{-1}\tilde{A}]\{\mu+\delta\}$$

$$\{\hat{\hat{C}}\} \simeq (\tilde{A}\ A)^{-1}\tilde{A}\ \mu + (\tilde{A}\ A)^{-1}\tilde{\Delta}\ \mu - (\tilde{A}\ A)^{-1}(\tilde{\Delta}A+\tilde{A}\Delta)(\tilde{A}\ A)^{-1}\tilde{A}\mu + (\tilde{A}\ A)^{-1}\tilde{A}\delta \quad \text{---(4)}$$

The first term in equation (4) is the least squares estimate of the coefficients obtained when errors are present only in the dependent variables and the model; the second and third terms give the effects of errors in the independent variables, with the second term indicating the combined effects of ill-conditionedness of the experiment and these errors; the fourth term the effect of errors in the dependent variables on the estimate of the coefficients.

Eq'n (4) can be used to find the confidence intervals for the estimates of the coefficients {C}.

Therefore

$$\hat{\hat{C}}_j \simeq \hat{C}_j + \Sigma_k \alpha_{jk}\delta_k + \Sigma_i \Sigma_k \beta_{ik}\Delta_{ik}$$

$$\sigma_{\hat{C}_j}^{\ 2} = \Sigma\alpha_{jk}^{\ 2}\ \sigma_{\delta_k}^{\ 2} + \Sigma\Sigma\beta_{ik}^{\ 2}\ \sigma_{\Delta_{ik}}^{\ 2}$$

The values of $\sigma_{\hat{C}_j}^2$ can be used to estimate confidence intervals for the parameters C_j as

$$C_j = \hat{C}_j \pm \sigma_{\hat{C}_j}$$

CHAPTER IV

RESULTS AND CONCLUSIONS

Computed Results:

The analysis of the equation

$$\mu_j = C_1L_{1j}+C_2L_{2j}+C_3L_{3j}+C_4a_j+C_5F_j+C_6v_j+C_7v_j^{\,2}+C_8a_j^{\,2}+C_9F_j^{\,2}+C_{10}a_jF_j+C_{11}a_jv_j+C_{12}v_jF_j$$

has revealed the following results:

	Estimate of the coefficient
C_1	- .472
C_2	- .353
C_3	- .255
C_4	+ .295
C_5	- .004
C_6	+ .004
C_7	- .0000006
C_8	- .026
C_9	+ .000004
C_{10}	+ .001
C_{11}	- .001
C_{12}	+ .000001

Table-8

The Standard Deviations for the Estimates

of the Co-efficients

	σ_{Δ}^{2}	σ_{δ}^{2}
c_1	8.748	0.046×10^{-1}
c_2	9.166	0.048×10^{-1}
c_3	8.857	0.045×10^{-1}
c_4	5.825	0.024×10^{-1}
c_5	0.022×10^{-2}	0.002×10^{-4}
c_6	0.485×10^{-3}	0.017×10^{-5}
c_7	0.147×10^{-8}	0.648×10^{-12}
c_8	0.2401	0.115×10^{-3}
c_9	0.605×10^{-9}	0.2404×10^{-11}
c_{10}	0.3324×10^{-4}	0.2788×10^{-7}
c_{11}	0.7495×10^{-4}	0.3188×10^{-7}
c_{12}	0.2538×10^{-8}	0.1865×10^{-11}

For further details of results appendix B may be referred.

Discussion And Conclusions:

On observing the computed values of the coefficients C_5, C_6, C_7, C_9, C_{10}, C_{11} and C_{12} which can be neglected being very small as compared to other coefficients, the resulting equation is written as

$$\mu_j = C_1L_{1j} + C_2L_{2j} + C_3L_{3j} + C_4a + C_8a^2$$

$$= -.472\ L_{1j} - .353\ L_{2j} - .255\ L_{3j} + .295\ a - .026\ a^2$$

As the radii of the indentors lie between

$$1.905 \leq a \leq 3.175$$

So

$$\mu_j = -.472L_{1j}-.353L_{2j}-.255L_{3j}+.295(a-1.905)+.5619-.026(a-1.905)^2-.02\ (3.810)(a-1.905)-.026(3.629)$$

$$= -.472L_{1j}-.353L_{2j}-.255L_{3j}+.295(a-1.905)-.026(a-1.905)^2-.101(a-1.905)+.4676$$

$$= (-.472+.4676)L_{1j}+(-.353+0.4676)L_{2j}+(-.255+0.4676)L_{3j}+0.194(a-1.905)-.026(a-1.905)^2$$

or

$$\mu_{\frac{gms\ sec}{cm^2}} = (-.0044)L_{1j}+(0.1146)L_{2j}+(0.2126)L_{3j}+0.194(a-1.905)-.026(a-1.905)^2$$

Converting this equation of μ with units as Centipoises by multiplying with a factor of (980.6 x 100), it becomes

$$\mu_{centipoises} = -431L_{1j}+11,238L_{2j}+20,848L_{3j}+19,024(a-1.905)-2540(a-1.905)^2$$

In the above equation, it is observed that the viscosity is varying with the radius of the indenter which does not agree with the theoretical results.

Moreover it is noticed from the above equation that the results will improve with highly viscous fluids and this will agree with the assumption in deriving the theoratical result, that the viscous effects are dominant. Further in deriving the theory, the indentation of an infinite half-space was considered while the experiments were conducted using very finite containers thereby introducing end effects.

Although efforts were made to keep the temperature of the room constant, the experiments should have been conducted in a precisely controlled environment to reduce the error due to temperature variations which changes the viscosity of the fluids to a great extent. Though the travel of indentation was fixed with the help of supports, even then variations were observed with different weights of the indentor.

Further, the author performed the experiments to find the viscosity of the same test samples using Brookfield Synchro-Lectric Viscometer. It was found that the results were quite satisfactory within reasonable limits of accuracy. But the viscometer could not be used at higher speeds with viscous fluids and is limited for use with fluids having viscosity up to 10^5 poises.

Recomendations For Future Work:

It is felt that if fluids of higher viscosity are used for the purpose of experiment there is every likelyhood that the results would be within very reasonable limits of accuracy. It is further observed that if larger containers are used, error will be minimized to a great extent.

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APPENDIX A

FORTRAN Program for Evaluation of Co-efficients by THE METHOD OF LEAST SQUARES.

```
$JOB            ACS,RUN=FREE,LINES=40,PAGES=20
C       INDENTOR  VISCOMETER  STUDIES
        IMPLICIT REAL*8(A-H,O-Z), INTEGER(I-N)
        DIMENSION N(81)
        DIMENSION D(81,12),Z(81),SG(81)
        DIMENSION A(81,12),F1(81),F2(81),F3(81)
        DIMENSION RWJ(81)
        DIMENSION R(81),W(81)
        DIMENSION V(4,81),VSQ(4,81),RVJ(4,81),VWJ(4,81)
        DIMENSION RSQ(81),WSQ(81)
  500   FORMAT(2F12.4,I4,I2)
  150   FORMAT(6F12.4,2I4)
  149   FORMAT(12X,5F12.4,2I4)
  700   FORMAT(I20,3F4.1,F6.2,2F8.2,F10.2,F6.2,F10.2,2F8.2,F10.2)
  800   FORMAT(1H1,//////////)
  100   FORMAT(3F4.1)
  600   FORMAT(6F12.4,I4,2I2)
        DO 200 J=1,81
  200   READ(1,100)F1(J),F2(J),F3(J)
        L1=1
        L2=2
        L3=3
        DO 151 J=1,81
        READ (1,149)     R(J),W(J),V(1,J),V(2,J),V(3,J),K,L1
        READ (1,150)VSQ(1,J),VSQ(2,J),VSQ(3,J),RSQ(J),WSQ(J),RWJ(J),K,L2
        READ (1,150)RVJ(1,J),RVJ(2,J),RVJ(3,J),VWJ(1,J),VWJ(2,J),VWJ(3,J),
       1L3
  151   CONTINUE
        MZ=81
        NZ=12
        DO 400 K=1,3
        DO 401 J=1,81
        A(J,1)=F1(J)
        A(J,2)=F2(J)
        A(J,3)=F3(J)
        A(J,4)=R(J)
        A(J,5)=W(J)
        A(J,6)=V(K,J)
        A(J,7)=VSQ(K,J)
        A(J,8)=RSQ(J)
```

```
      A(J,9)=WSQ(J)
      A(J,10)=RWJ(J)
      A(J,11)=RVJ(K,J)
      A(J,12)=VWJ(K,J)
      Z(J)=A(J,5)/(8.0*A(J,4)*A(J,6))
  401 CONTINUE
      DO 101 J=1,79,3
      SG(J)=DSQRT((Z(J)-((Z(J)+Z(J+1)+Z(J+2))/3.0))**2+(Z(J+1)-((Z(J)+Z(
     1J+1)+Z(J+2))/3.0))**2+(Z(J+2)-((Z(J)+Z(J+1)+Z(J+2))/3.0))**2)/
     2DSQRT(.3D+1)
      SG(J+1)=SG(J)
      SG(J+2)=SG(J)
      D(J,6)=DSQRT((A(J,6)-((A(J,6)+A(J+1,6)+A(J+2,6))/3.0))**2+
     1(A(J+1,6)-((A(J,6)+A(J+1,6)+A(J+2,6))/3.0))**2+(A(J+2,6)-((A(J,6)
     2+A(J+2,6))/3.0))**2)/DSQRT(.3D+1)
      D(J+1,6)=D(J,6)
      D(J+2,6)=D(J,6)
  101 CONTINUE
      DO 410 J=1,81
      D(J,1)=0.0
      D(J,2)=0.0
      D(J,3)=0.0
      D(J,4)=0.0
      D(J,5)=0.0
      D(J,7)=D(J,6)**2
      D(J,8)=0.0
      D(J,9)=0.0
      D(J,10)=0.0
      D(J,11)=D(J,6)*A(J,4)
      D(J,12)=D(J,6)*A(J,5)
      N1=1
      N2=2
  410 CONTINUE
      CALL ERRORS(MZ,NZ,A,D,Z,SG)
  400 CONTINUE
      STOP
      END
```

```
      SUBROUTINE ERRORS(M,N,X,SIGX,Z,SIGZ)
      IMPLICIT REAL*8(A-H,O-Z), INTEGER(I-N)
      DIMENSION ZETA(81),ZETASQ(81)
      DIMENSION RO(81,12),ROSQ(81,12)
      DIMENSION A1(81,12),A2(81,12),A3(81,12),ASUM(81,12)
C     ***NOTE**** ALPHA(I,J) MUST BE DIMENSIONED (N+1)*(2N+2)
      DIMENSION ALPHA(12,24)
      DIMENSION SAVE(12,12),CHECK(12,12)
      DIMENSION X(81,12),SIGX(81,12),Z(81),SIGZ(81)
      DIMENSION AHAT(12),GAMMA(81)
      DIMENSION XTRAN(12,81),BETA(12,81)
      DIMENSION ASQ(81,12),BSQ(12,81)
      DIMENSION SUM1(12),SUM2(12)
C     M IS THE NUMBER OF EXPERIMENTS
C     N IS THE NUMBER OF INDEPENDENT VARIABLES
C     X(I,J) AND SIGX(I,J) ARE (M)*(N+1) ARRAYS
C     Z(I) AND SIGZ(I) ARE COLUMNS OF (M) ELEMENTS EACH
C     ASQ(I,J) IS AN (M)*(N+1) ARRAY
C     BSQ(I,J) IS AN (N+1)*(M) ARRAY
C     AHAT(I) IS A COLUMN OF (N+1) ELEMENTS
C     XTRAN(I,J) AND BETA(I,J) ARE (N+1)*(M) ARRAYS
C     ALPHA(I,J) IS AN (N+1)*(N+1) ARRAY  ****(SEE NOTE ABOVE)****
C     GAMMA(I) IS A COLUMN OF (M) ELEMENTS
C     CHECK(I,J) AND SAVE(I,J) ARE (N+1)*(N+1) ARRAYS
C     A1(I,J),A2(I,J),A3(I,J), AND ASUM(I,J) ARE (M)*(N+1) ARRAYS
C     SUM1(I) AND SUM2(I) ARE COLUMNS OF N1 ELEMENTS EACH
C     ZETA(J) AND ZETASQ(J) ARE COLUMNS OF M ELEMENTS EACH
C     RO(J,K) AND ROSQ(J,K) ARE (M)*(N+1) ARRAYS
C     IX(J) IS A COLUMN OF (N+1) ELEMENTS
  101 FORMAT(4I5)
  102 FORMAT(4F10.5)
  103 FORMAT(5F10.5)
  104 FORMAT(4E16.8)
  105 FORMAT(5E16.8)
  106 FORMAT(1H1)
  107 FORMAT(1H0)
  108 FORMAT(8F10.5)
  109 FORMAT(6E16.8)
  110 FORMAT(12E11.3)
  111 FORMAT(10X,2HI=,I3)
```

```
  112 FORMAT(11X,3I4,4I5,29X,I4)
      N1=N
      WRITE(3,106)
      DO 2000 I=1,M
 2000 WRITE(3,109) (X(I,J),J=1,N1),Z(I)
      WRITE(3,106)
      DO 2001 I=1,M
 2001 WRITE(3,109) (SIGX(I,J),J=1,N1),SIGZ(I)
      WRITE(3,106)
      DO 203 I=1,M
      DO 203 J=1,N1
  203 XTRAN(J,I)=X(I,J)
      CALL MATMUL(XTRAN,X,ALPHA,N1,M,N1)
      DO 200 I=1,N1
      DO 200 J=1,N1
  200 SAVE(I,J)=ALPHA(I,J)
      N2=2*N1
      CALL INVRS(ALPHA,N1,N2)
      CALL MATMUL(ALPHA,XTRAN,BETA,N1,N1,M)
      CALL MATCOL(BETA,Z,AHAT,N1,M)
      CALL MATCOL(X,AHAT,GAMMA,M,N1)
      CALL MATMUL(ALPHA,SAVE,CHECK,N1,N1,N1)
C     CHECK(I,J) SHOULD BE THE UNIT MATRIX I
      WRITE(3,110) ((CHECK(I,J),J=1,N1),I=1,N1)
      WRITE(3,107)
      DO 206 I=1,N1
  206 WRITE(3,104) AHAT(I)
      WRITE(3,106)
      DO 205 I=1,N1
      DO 204 K=1,M
      DO 204 J=1,N1
      B1=Z(K)*ALPHA(I,J)
      B2=GAMMA(K)*ALPHA(I,J)
      B3=AHAT(J)*BETA(I,K)
      B4=B1-B2-B3
      ASUM(K,J)=B4
      ASQ(K,J)=B4*B4
      B5=BETA(J,K)
      BSQ(J,K)=B5*B5
      A1(K,J)=B1
```

```
      A2(K,J)=B2
  204 A3(K,J)=B3
C     AT THIS POINT FOR EACH (I) THERE ARE FOUR (M)*(N+1) ARRAYS
C     A1, A2, A3, AND ASUM AS WELL AS THE (N+1)*(N+1) ARRAY BETA.
      WRITE(3,111)  I
      WRITE(3,110) ((A1(L,J),J=1,N1),L=1,M)
      WRITE(3,106)
      WRITE(3,111)  I
      WRITE(3,110) ((A2(L,J),J=1,N1),L=1,M)
      WRITE(3,106)
      WRITE(3,111)  I
      WRITE(3,110) ((A3(L,J),J=1,N1),L=1,M)
      WRITE(3,106)
      WRITE(3,111)  I
      WRITE(3,110) ((ASUM(L,J),J=1,N1),L=1,M)
      WRITE(3,106)
      WRITE(3,111)  I
      WRITE(3,110) ((BETA(L,J),L=1,N1),J=1,M)
      WRITE(3,106)
      WRITE(3,111)  I
      DO 207 L=1,M
  207 WRITE(3,110) (ASQ(L,J),J=1,N1)
      WRITE(3,106)
      WRITE(3,111)  I
      DO 208 J=1,M
  208 WRITE(3,110) (BSQ(L,J),L=1,N1)
      WRITE(3,107)
      SUM1(I)=0.0
      SUM2(I)=0.0
      DO 210 L=1,M
      DO 209 J=1,N1
      C1=SIGX(L,J)
  209 SUM1(I)=SUM1(I)+ASQ(L,J)*C1*C1
      C2=SIGZ(L)
  210 SUM2(I)=SUM2(I)+BSQ(I,L)*C2*C2
      WRITE(3,105) SUM1(I),SUM2(I)
  205 WRITE(3,106)
      EHAT1=0.0
      EHAT2=0.0
      DO 300 J=1,M
```

```
      EHAT1=EHAT1+Z(J)*Z(J)
      EHAT2=EHAT2+Z(J)*GAMMA(J)
  300 ZETA(J)=(Z(J)-GAMMA(J))*2.0
      ENORM=EHAT1-EHAT2
      DO 301 J=1,M
      DO 302 K=1,N1
      DUMMY=ZETA(J)*AHAT(K)
      RO(J,K)=DUMMY
  302 ROSQ(J,K)=DUMMY*DUMMY
      DUMMY=ZETA(J)
  301 ZETASQ(J)=DUMMY*DUMMY
      SUM3=0.0
      SUM4=0.0
      DO 303 I=1,M
      DO 304 J=1,N1
      C1=SIGX(I,J)
  304 SUM3=SUM3+ROSQ(I,J)*C1*C1
      C2=SIGZ(I)
  303 SUM4=SUM4+ZETASQ(I)*C2*C2
      DO 305 L=1,M
  305 WRITE(3,110) (ROSQ(L,J),J=1,N1)
      WRITE(3,106)
      DO 306 J=1,M
  306 WRITE(3,105) ZETASQ(J)
      WRITE(3,107)
      WRITE(3,105) ENORM,EHAT1,EHAT2,SUM3,SUM4
      WRITE(3,106)
      STOP
      END
```

```
      SUBROUTINE INVRS(A,N,N2)
      IMPLICIT REAL*8(A-H,O-Z), INTEGER(I-N)
C     THIS SUBROUTINE INVERTS MATRIX A OF ORDER N
C     BY GAUSS-SIEDEL REDUCTION.
C     THE MATRIX A MUST BE OF DIMENSION (N)*(2N) IN THE CALLING PROGRAM.
C     THE ARGUMENT N2 MUST HAVE A VALUE OF 2*N IN THE CALLING PROGRAM.
      DIMENSION A(N,N2)
    2 FORMAT(2(//20X,10HNO INVERSE //))
C     AUGMENTING THE MATRIX A BY AN IDENTITY MATRIX
      NN=N+N
      DO 20 I=1,N
      IN=I+N
      DO 10 J=1,N
      JN=J+N
   10 A(I,JN)=0.
   20 A(I,IN)=1.
C     THE REDUCTION PROCESS STARTS HERE
      DO 100 M=1,N
   30 DIV=A(M,M)
      IF(DIV.EQ.0.0) GO TO 70
      DO 40 J=1,NN
   40 A(M,J)=A(M,J)/DIV
      DO 60 I=1,N
      IF(I.EQ.M) GO TO 60
      AIM=A(I,M)
      DO 50 J=1,NN
   50 A(I,J)=A(I,J)-AIM*A(M,J)
   60 CONTINUE
      GO TO 100
   70 DO 90 I=M,N
      IF(A(I,M).EQ.0.0) GO TO 90
C     DO LOOP 80 EFFECTS INTERCHANGE OF THE ITH AND MTH ROWS
      DO 80 J=1,NN
      DUMY=A(I,J)
      A(I,J)=A(M,J)
   80 A(M,J)=DUMY
      GO TO 30
   90 CONTINUE
      WRITE(3,2)
      GO TO 120
```

```
  100 CONTINUE
C     TRANSFERING INVERSE OF A IN THE PLACE OF A
      DO 110 I=1,N
      DO 110 J=1,N
      JN=J+N
  110 A(I,J)=A(I,JN)
  120 RETURN
      END
```

```
      SUBROUTINE MATMUL(A,B,C,M,N,L)
      IMPLICIT REAL*8(A-H,O-Z), INTEGER(I-N)
C     THIS SUBROUTINE FORMS THE MATRIX PRODUCT AB=C WHERE A IS AN
C     (M)*(N) ARRAY, B IS AN (N)*(L) ARRAY AND THE PRODUCT C IS
C     AN (M)*(L) ARRAY.
      DIMENSION A(M,N),B(N,L),C(M,L)
      DO 100 I=1,M
      DO 100 K=1,L
      C(I,K)=0.0
      DO 100 J=1,N
  100 C(I,K)=C(I,K)+A(I,J)*B(J,K)
      RETURN
      END
```

```
      SUBROUTINE MATCOL(A,B,C,M,N)
      IMPLICIT REAL*8(A-H,O-Z), INTEGER(I-N)
C     THIS SUBROUTINE FORMS THE MATRIX PRODUCT AB=C WHERE A IS AN
C     (M)*(N) ARRAY, B IS A COLUMN OF N ELEMENTS AND THE PRODUCT C
C     IS A COLUMN OF M ELEMENTS.
      DIMENSION A(M,N),B(N),C(M)
      DO 100 I=1,M
      C(I)=0.0
      DO 100 J=1,N
  100 C(I)=C(I)+A(I,J)*B(J)
      RETURN
      END
```

APPENDIX B

INPUT DATA FOR THE COMPUTER PROGRAM:

Notation:

L1	Fluid (1)
L2	Fluid (2)
L3	Fluid (3)
R	Radius Of Indentor
W	Weight Of Indentor
V	Velocity Of Indentation
RW	Radius x Weight
RV	Radius x Velocity
VW	Velocity x Weight

L1	L2	L3	R	W	V1	$V1^2$	R^2	W^2	RW	RV1	V1W
1.0	0.0	0.0	1.90	103.50	192.00	36863.99	3.63	10712.25	197.17	365.76	19871.99
1.0	0.0	0.0	1.90	103.50	184.62	34082.82	3.63	10712.25	197.17	351.69	19107.68
1.0	0.0	0.0	1.90	103.50	184.62	34082.82	3.63	10712.25	197.17	351.69	19107.68
0.0	1.0	0.0	1.90	103.50	106.67	11377.77	3.63	10712.25	197.17	203.20	11039.99
0.0	1.0	0.0	1.90	103.50	106.67	11377.77	3.63	10712.25	197.17	203.20	11039.99
0.0	1.0	0.0	1.90	103.50	104.35	10888.46	3.63	10712.25	197.17	198.78	10799.99
0.0	0.0	1.0	1.90	103.50	73.85	5453.25	3.63	10712.25	197.17	140.68	7643.07
0.0	0.0	1.0	1.90	103.50	67.61	4570.52	3.63	10712.25	197.17	128.79	6997.18
0.0	0.0	1.0	1.90	103.50	71.64	5132.54	3.63	10712.25	197.17	136.48	7414.92
1.0	0.0	0.0	2.54	103.50	100.00	9999.99	6.45	10712.25	262.89	254.00	10349.99
1.0	0.0	0.0	2.54	103.50	97.96	9596.00	6.45	10712.25	262.89	248.82	10138.77
1.0	0.0	0.0	2.54	103.50	97.96	9596.00	6.45	10712.25	262.89	248.82	10138.77
1.0	0.0	0.0	3.18	103.50	35.29	1245.67	10.08	10712.25	328.61	112.06	3652.94
1.0	0.0	0.0	3.18	103.50	37.21	1384.53	10.08	10712.25	328.61	118.14	3851.16
1.0	0.0	0.0	3.18	103.50	36.92	1363.31	10.08	10712.25	328.61	117.23	3821.54
1.0	0.0	0.0	1.90	138.50	228.57	52244.86	3.63	19182.25	263.84	435.43	31657.13
1.0	0.0	0.0	1.90	138.50	228.57	52244.91	3.63	19182.25	263.84	435.43	31657.14
1.0	0.0	0.0	1.90	138.50	218.18	47603.30	3.63	19182.25	263.84	415.64	30218.17
1.0	0.0	0.0	1.90	173.50	240.00	57600.00	3.63	30102.25	330.52	457.20	41640.00
1.0	0.0	0.0	1.90	173.50	240.00	57600.00	3.63	30102.25	330.52	457.20	41640.00
1.0	0.0	0.0	1.90	173.50	240.00	57600.00	3.63	30102.25	330.52	457.20	41640.00
0.0	1.0	0.0	2.54	103.50	45.28	2050.55	6.45	10712.25	262.89	115.02	4686.79
0.0	1.0	0.0	2.54	103.50	46.15	2130.18	6.45	10712.25	262.89	117.23	4776.92
0.0	1.0	0.0	2.54	103.50	50.53	2552.91	6.45	10712.25	262.89	128.34	5229.47
0.0	1.0	0.0	3.18	103.50	17.52	306.89	10.08	10712.25	328.61	55.62	1813.14
0.0	1.0	0.0	3.18	103.50	19.28	371.61	10.08	10712.25	328.61	61.20	1995.18
0.0	1.0	0.0	3.18	103.50	17.78	316.05	10.08	10712.25	328.61	56.44	1840.00

0.0	0.0	1.0	2.54	103.50	32.43	1051.86	6.45	10712.25	262.89	82.38	3356.76
0.0	0.0	1.0	2.54	103.50	32.43	1051.86	6.45	10712.25	262.89	82.38	3356.75
0.0	0.0	1.0	2.54	103.50	32.21	1037.79	6.45	10712.25	262.89	81.83	3334.23
0.0	0.0	1.0	3.18	103.50	11.97	143.28	10.08	10712.25	328.61	38.01	1238.90
0.0	0.0	1.0	3.18	103.50	12.57	157.89	10.08	10712.25	328.61	39.90	1300.52
0.0	0.0	1.0	3.18	103.50	11.43	130.61	10.08	10712.25	328.61	36.29	1182.86
0.0	1.0	0.0	2.54	138.50	69.57	4839.32	6.45	19182.25	351.79	176.70	9634.78
0.0	1.0	0.0	2.54	138.50	66.67	4444.44	6.45	19182.25	351.79	169.33	9233.33
0.0	1.0	0.0	2.54	138.50	67.61	4570.52	6.45	19182.25	351.79	171.72	9363.38
0.0	1.0	0.0	2.54	173.50	90.57	8202.20	6.45	30102.25	440.69	230.04	15713.20
0.0	1.0	0.0	2.54	173.50	84.21	7091.41	6.45	30102.25	440.69	213.89	14610.52
0.0	1.0	0.0	2.54	173.50	85.71	7346.94	6.45	30102.25	440.69	217.71	14871.42
0.0	1.0	0.0	3.18	138.50	26.09	680.53	10.08	19182.25	439.74	82.83	3613.04
0.0	1.0	0.0	3.18	138.50	23.65	559.10	10.08	19182.25	439.74	75.07	3274.87
0.0	1.0	0.0	3.18	138.50	24.12	581.80	10.08	19182.25	439.74	76.58	3340.70
0.0	1.0	0.0	3.18	173.50	33.33	1111.11	10.08	30102.25	550.86	105.83	5783.33
0.0	1.0	0.0	3.18	173.50	30.38	922.93	10.08	30102.25	550.86	96.46	5270.88
0.0	1.0	0.0	3.18	173.50	32.21	1037.79	10.08	30102.25	550.86	102.28	5589.26
0.0	0.0	1.0	2.54	138.50	42.11	1772.85	6.45	19182.25	351.79	106.95	5831.57
0.0	0.0	1.0	2.54	138.50	44.86	2012.40	6.45	19182.25	351.79	113.94	6213.08
0.0	0.0	1.0	2.54	138.50	50.53	2552.91	6.45	19182.25	351.79	128.34	6997.89
0.0	0.0	1.0	3.18	138.50	16.90	285.66	10.08	19182.25	439.74	53.66	2340.84
0.0	0.0	1.0	3.18	138.50	15.58	242.87	10.08	19182.25	439.74	49.48	2158.44
0.0	0.0	1.0	3.18	138.50	16.67	277.78	10.08	19182.25	439.74	52.92	2308.33
0.0	0.0	1.0	3.18	173.50	21.05	443.21	10.08	30102.25	550.86	66.84	3652.63
0.0	0.0	1.0	3.18	173.50	20.00	400.00	10.08	30102.25	550.86	63.50	3470.00
0.0	0.0	1.0	3.18	173.50	20.51	420.77	10.08	30102.25	550.86	65.13	3558.97

1.0	0.0	0.0	2.54	138.50	154.84	23975.03	6.45	19182.25	351.79	393.29	21445.16
1.0	0.0	0.0	2.54	138.50	137.14	18808.15	6.45	19182.25	351.79	348.34	18994.28
1.0	0.0	0.0	2.54	138.50	133.33	17777.77	6.45	19182.25	351.79	338.67	18466.66
1.0	0.0	0.0	2.54	173.50	171.43	29387.76	6.45	30102.25	440.69	435.43	29742.85
1.0	0.0	0.0	2.54	173.50	171.43	29387.74	6.45	30102.25	440.69	435.43	29742.85
1.0	0.0	0.0	2.54	173.50	165.52	27395.95	6.45	30102.25	440.69	420.41	28717.23
1.0	0.0	0.0	3.18	138.50	49.48	2448.72	10.08	19182.25	439.74	157.11	6853.60
1.0	0.0	0.0	3.18	138.50	55.17	3043.99	10.08	19182.25	439.74	175.17	7641.38
1.0	0.0	0.0	3.18	138.50	52.17	2722.11	10.08	19182.25	439.74	165.65	7226.08
1.0	0.0	0.0	3.18	173.50	64.86	4207.45	10.08	30102.25	550.86	205.95	11254.05
1.0	0.0	0.0	3.18	173.50	82.76	6848.98	10.08	30102.25	550.86	262.76	14358.61
1.0	0.0	0.0	3.18	173.50	66.67	4444.44	10.08	30102.25	550.86	211.67	11566.66
0.0	1.0	0.0	1.90	138.50	154.84	23975.03	3.63	19182.25	263.84	294.97	21445.16
0.0	1.0	0.0	1.90	138.50	154.84	23975.03	3.63	19182.25	263.84	294.97	21445.16
0.0	1.0	0.0	1.90	138.50	145.45	21157.01	3.63	19182.25	263.84	277.09	20145.44
0.0	1.0	0.0	1.90	173.50	184.62	34082.85	3.63	30102.25	330.52	351.69	32030.77
0.0	1.0	0.0	1.90	173.50	192.00	36863.99	3.63	30102.25	330.52	365.76	33311.99
0.0	1.0	0.0	1.90	173.50	192.00	36863.99	3.63	30102.25	330.52	365.76	33311.99
0.0	0.0	1.0	1.90	138.50	100.00	9999.99	3.63	19182.25	263.84	190.50	13849.99
0.0	0.0	1.0	1.90	138.50	100.00	9999.99	3.63	19182.25	263.84	190.50	13849.99
0.0	0.0	1.0	1.90	138.50	109.09	11900.82	3.63	19182.25	263.84	207.82	15109.08
0.0	0.0	1.0	1.90	173.50	133.33	17777.76	3.63	30102.25	330.52	254.00	23133.32
0.0	0.0	1.0	1.90	173.50	137.14	18808.15	3.63	30102.25	330.52	261.26	23794.28
0.0	0.0	1.0	1.90	173.50	133.33	17777.76	3.63	30102.25	330.52	254.00	23133.32
0.0	0.0	1.0	2.54	173.50	55.17	3043.99	6.45	30102.25	440.69	140.14	9572.41
0.0	0.0	1.0	2.54	173.50	57.83	3344.46	6.45	30102.25	440.69	146.89	10033.72
0.0	0.0	1.0	2.54	173.50	51.06	2607.51	6.45	30102.25	440.69	129.70	8859.57

L1	L2	L3	R	W	V2	$V2^2$	R^2	W^2	RW	RV2	V2W
1.0	0.0	0.0	1.90	103.50	200.00	40000.00	3.63	10712.25	197.17	381.00	20700.00
1.0	0.0	0.0	1.90	103.50	200.00	40000.00	3.63	10712.25	197.17	381.00	20700.00
1.0	0.0	0.0	1.90	103.50	218.18	47603.30	3.63	10712.25	197.17	415.64	22581.81
0.0	1.0	0.0	1.90	103.50	100.00	9999.99	3.63	10712.25	197.17	190.50	10349.99
0.0	1.0	0.0	1.90	103.50	100.00	10000.00	3.63	10712.25	197.17	190.50	10350.00
0.0	1.0	0.0	1.90	103.50	96.00	9216.00	3.63	10712.25	197.17	182.88	9936.00
0.0	0.0	1.0	1.90	103.50	67.61	4570.52	3.63	10712.25	197.17	128.79	6997.18
0.0	0.0	1.0	1.90	103.50	63.16	3988.91	3.63	10712.25	197.17	120.32	6536.83
0.0	0.0	1.0	1.90	103.50	65.75	4323.51	3.63	10712.25	197.17	125.26	6805.48
1.0	0.0	0.0	2.54	103.50	97.96	9596.00	6.45	10712.25	262.89	248.82	10138.77
1.0	0.0	0.0	2.54	103.50	94.12	8858.13	6.45	10712.25	262.89	239.06	9741.17
1.0	0.0	0.0	2.54	103.50	96.00	9216.00	6.45	10712.25	262.89	243.84	9936.00
1.0	0.0	0.0	3.18	103.50	33.10	1095.84	10.08	10712.25	328.61	105.10	3426.21
1.0	0.0	0.0	3.18	103.50	34.53	1192.48	10.08	10712.25	328.61	109.64	3574.10
1.0	0.0	0.0	3.18	103.50	34.53	1192.48	10.08	10712.25	328.61	109.64	3574.10
1.0	0.0	0.0	1.90	138.50	282.35	79723.06	3.63	19182.25	263.84	537.88	39105.65
1.0	0.0	0.0	1.90	138.50	266.67	71111.00	3.63	19182.25	263.84	508.00	36933.30
1.0	0.0	0.0	1.90	138.50	266.67	71111.00	3.63	19182.25	263.84	508.00	36933.30
1.0	0.0	0.0	1.90	173.50	320.00	102400.00	3.63	30102.25	330.52	609.60	55520.00
1.0	0.0	0.0	1.90	173.50	266.67	71111.13	3.63	30102.25	330.52	508.00	46266.67
1.0	0.0	0.0	1.90	173.50	300.00	90000.00	3.63	30102.25	330.52	571.50	52050.00
0.0	1.0	0.0	2.54	103.50	44.44	1975.31	6.45	10712.25	262.89	112.89	4600.00
0.0	1.0	0.0	2.54	103.50	43.64	1904.13	6.45	10712.25	262.89	110.84	4516.36
0.0	1.0	0.0	2.54	103.50	47.52	2258.60	6.45	10712.25	262.89	120.71	4918.81
0.0	1.0	0.0	3.18	103.50	16.27	264.75	10.08	10712.25	328.61	51.66	1684.07
0.0	1.0	0.0	3.18	103.50	17.52	306.89	10.08	10712.25	328.61	55.62	1813.14
0.0	1.0	0.0	3.18	103.50	16.61	275.86	10.08	10712.25	328.61	52.73	1719.03

0.0	0.0	1.0	2.54	103.50	30.38	922.93	6.45	10712.25	262.89	77.16	3144.30
0.0	0.0	1.0	2.54	103.50	30.77	946.75	6.45	10712.25	262.89	76.15	3184.61
0.0	0.0	1.0	2.54	103.50	30.57	934.72	6.45	10712.25	262.89	77.66	3164.33
0.0	0.0	1.0	3.18	103.50	10.31	116.87	10.08	10712.25	328.61	34.32	1118.92
0.0	0.0	1.0	3.18	103.50	11.40	129.99	10.08	10712.25	328.61	36.20	1180.05
0.0	0.0	1.0	3.18	103.50	10.39	107.94	10.08	10712.25	328.61	32.99	1075.32
0.0	1.0	0.0	2.54	138.50	64.86	4207.45	6.45	19182.25	351.79	164.76	8983.78
0.0	1.0	0.0	2.54	138.50	63.16	3988.92	6.45	19182.25	351.79	160.42	8747.36
0.0	1.0	0.0	2.54	138.50	64.00	4096.00	6.45	19182.25	351.79	162.56	8864.00
0.0	1.0	0.0	2.54	173.50	84.21	7091.41	6.45	30102.25	440.69	213.89	14610.52
0.0	1.0	0.0	2.54	173.50	76.19	5805.00	6.45	30102.25	440.69	193.52	13219.05
0.0	1.0	0.0	2.54	173.50	80.00	6400.00	6.45	30102.25	440.69	203.20	13880.00
0.0	1.0	0.0	3.18	138.50	24.00	576.00	10.08	19182.25	439.74	76.20	3324.00
0.0	1.0	0.0	3.18	138.50	22.02	484.81	10.08	19182.25	439.74	69.91	3049.54
0.0	1.0	0.0	3.18	138.50	22.22	493.83	10.08	19182.25	439.74	70.56	3077.78
0.0	1.0	0.0	3.18	173.50	30.00	900.00	10.08	30102.25	550.86	95.25	5205.00
0.0	1.0	0.0	3.18	173.50	27.91	778.80	10.08	30102.25	550.86	88.60	4841.86
0.0	1.0	0.0	3.18	173.50	29.27	856.63	10.08	30102.25	550.86	92.93	5078.04
0.0	0.0	1.0	2.54	138.50	39.34	1547.97	6.45	19182.25	351.79	99.93	5449.17
0.0	0.0	1.0	2.54	138.50	43.24	1869.98	6.45	19182.25	351.79	109.84	5989.19
0.0	0.0	1.0	2.54	138.50	43.24	1869.98	6.45	19182.25	351.79	109.84	5989.18
0.0	0.0	1.0	3.18	138.50	15.38	236.69	10.08	19182.25	439.74	48.85	2130.77
0.0	0.0	1.0	3.18	138.50	14.12	199.31	10.08	19182.25	439.74	44.82	1955.29
0.0	0.0	1.0	3.18	138.50	15.38	236.69	10.08	19182.25	439.74	48.85	2130.77
0.0	0.0	1.0	3.18	173.50	19.35	374.61	10.08	30102.25	550.86	61.45	3358.06
0.0	0.0	1.0	3.18	173.50	18.46	340.83	10.08	30102.25	550.86	58.62	3203.07
0.0	0.0	1.0	3.18	173.50	18.75	351.56	10.08	30102.25	550.86	59.53	3253.13

1.0	0.0	0.0	2.54	138.50	145.45	21157.01	6.45	19182.25	351.79	369.45	20145.44
1.0	0.0	0.0	2.54	138.50	133.33	17777.76	6.45	19182.25	351.79	338.67	18466.66
1.0	0.0	0.0	2.54	138.50	129.73	16829.79	6.45	19182.25	351.79	329.51	17967.56
1.0	0.0	0.0	2.54	173.50	165.52	27395.94	6.45	30102.25	440.69	420.41	28717.23
1.0	0.0	0.0	2.54	173.50	177.78	31604.93	6.45	30102.25	440.69	451.56	30844.44
1.0	0.0	0.0	2.54	173.50	177.78	31604.93	6.45	30102.25	440.69	451.56	30844.44
1.0	0.0	0.0	3.18	138.50	45.28	2050.55	10.08	19182.25	439.74	143.77	6271.69
1.0	0.0	0.0	3.18	138.50	48.98	2399.00	10.08	19182.25	439.74	155.51	6783.67
1.0	0.0	0.0	3.18	138.50	47.52	2258.60	10.08	19182.25	439.74	150.89	6582.18
1.0	0.0	0.0	3.18	173.50	60.00	3600.00	10.08	30102.25	550.86	190.50	10410.00
1.0	0.0	0.0	3.18	173.50	65.75	4323.52	10.08	30102.25	550.86	208.77	11408.22
1.0	0.0	0.0	3.18	173.50	63.16	3988.92	10.08	30102.25	550.86	200.53	10957.89
0.0	1.0	0.0	1.90	138.50	150.00	22499.98	3.63	19182.25	263.84	285.75	20774.99
0.0	1.0	0.0	1.90	138.50	137.14	18808.15	3.63	19182.25	263.84	261.26	18994.28
0.0	1.0	0.0	1.90	138.50	141.18	19930.79	3.63	19182.25	263.84	268.94	19552.93
0.0	1.0	0.0	1.90	173.50	184.62	34082.82	3.63	30102.25	330.52	351.69	32030.75
0.0	1.0	0.0	1.90	173.50	192.00	36863.99	3.63	30102.25	330.52	365.76	33311.99
0.0	1.0	0.0	1.90	173.50	192.00	36863.99	3.63	30102.25	330.52	365.76	33311.99
0.0	0.0	1.0	1.90	138.50	87.27	7616.53	3.63	19182.25	263.84	166.25	12087.26
0.0	0.0	1.0	1.90	138.50	90.57	8202.21	3.63	19182.25	263.84	172.53	12543.39
0.0	0.0	1.0	1.90	138.50	100.00	9999.99	3.63	19182.25	263.84	190.50	13849.99
0.0	0.0	1.0	1.90	173.50	120.00	14400.00	3.63	30102.25	330.52	228.60	20820.00
0.0	0.0	1.0	1.90	173.50	117.07	13706.12	3.63	30102.25	330.52	223.02	20312.19
0.0	0.0	1.0	1.90	173.50	117.07	13706.12	3.63	30102.25	330.52	223.02	20312.19
0.0	0.0	1.0	2.54	173.50	50.00	2500.00	6.45	30102.25	440.69	127.00	8674.99
0.0	0.0	1.0	2.54	173.50	52.75	2782.27	6.45	30102.25	440.69	133.98	9151.65
0.0	0.0	1.0	2.54	173.50	58.54	3426.53	6.45	30102.25	440.69	148.68	10156.08

L1	L2	L3	R	W	V3	$V3^2$	R^2	W^2	RW	RV3	V3W
1.0	0.0	0.0	1.90	103.50	177.78	31604.91	3.63	10712.25	197.17	338.67	18399.99
1.0	0.0	0.0	1.90	103.50	177.78	31604.93	3.63	10712.25	197.17	338.67	18400.00
1.0	0.0	0.0	1.90	103.50	177.78	31604.93	3.63	10712.25	197.17	338.67	18400.00
0.0	1.0	0.0	1.90	103.50	88.89	7901.24	3.63	10712.25	197.17	169.33	9200.00
0.0	1.0	0.0	1.90	103.50	87.27	7616.54	3.63	10712.25	197.17	166.25	9032.73
0.0	1.0	0.0	1.90	103.50	88.89	7901.22	3.63	10712.25	197.17	169.33	9199.99
0.0	0.0	1.0	1.90	103.50	60.76	3691.70	3.63	10712.25	197.17	115.75	6288.60
0.0	0.0	1.0	1.90	103.50	55.81	3115.20	3.63	10712.25	197.17	106.33	5776.75
0.0	0.0	1.0	1.90	103.50	59.26	3511.65	3.63	10712.25	197.17	112.89	6133.32
1.0	0.0	0.0	2.54	103.50	84.21	7091.42	6.45	10712.25	262.89	213.89	8715.80
1.0	0.0	0.0	2.54	103.50	80.00	6400.01	6.45	10712.25	262.89	203.20	8280.01
1.0	0.0	0.0	2.54	103.50	82.76	6848.97	6.45	10712.25	262.89	210.21	8565.50
1.0	0.0	0.0	3.18	103.50	27.75	769.82	10.08	10712.25	328.61	88.09	2871.67
1.0	0.0	0.0	3.18	103.50	28.92	836.11	10.08	10712.25	328.61	91.81	2992.77
1.0	0.0	0.0	3.18	103.50	31.17	971.49	10.08	10712.25	328.61	98.96	3225.97
1.0	0.0	0.0	1.90	138.50	240.00	57600.00	3.63	19182.25	263.84	457.20	33240.00
1.0	0.0	0.0	1.90	138.50	240.00	57600.00	3.63	19182.25	263.84	457.20	33240.00
1.0	0.0	0.0	1.90	138.50	228.57	52244.91	3.63	19182.25	263.84	435.43	31657.14
1.0	0.0	0.0	1.90	173.50	282.35	79723.06	3.63	30102.25	330.52	537.88	48988.20
1.0	0.0	0.0	1.90	173.50	342.86	117550.60	3.63	30102.25	330.52	653.14	59485.63
1.0	0.0	0.0	1.90	173.50	282.35	79723.06	3.63	30102.25	330.52	537.88	48988.20
0.0	1.0	0.0	2.54	103.50	38.40	1474.56	6.45	10712.25	262.89	97.54	3974.40
0.0	1.0	0.0	2.54	103.50	38.10	1451.25	6.45	10712.25	262.89	96.76	3942.86
0.0	1.0	0.0	2.54	103.50	42.48	1804.37	6.45	10712.25	262.89	107.89	4396.45
0.0	1.0	0.0	3.18	103.50	13.83	191.35	10.08	10712.25	328.61	43.92	1431.70
0.0	1.0	0.0	3.18	103.50	15.00	225.00	10.08	10712.25	328.61	47.63	1552.50
0.0	1.0	0.0	3.18	103.50	13.99	195.84	10.08	10712.25	328.61	44.43	1448.40

0.0	0.0	1.0	2.54	103.50	26.97	727.18	6.45	10712.25	262.89	68.49	2791.01
0.0	0.0	1.0	2.54	103.50	27.43	752.33	6.45	10712.25	262.89	69.67	2838.85
0.0	0.0	1.0	2.54	103.50	27.12	735.42	6.45	10712.25	262.89	68.88	2806.78
0.0	0.0	1.0	3.18	103.50	8.73	76.17	10.08	10712.25	328.61	27.71	903.27
0.0	0.0	1.0	3.18	103.50	9.43	88.93	10.08	10712.25	328.61	29.94	976.03
0.0	0.0	1.0	3.18	103.50	8.96	80.20	10.08	10712.25	328.61	28.43	926.87
0.0	1.0	0.0	2.54	138.50	58.54	3426.52	6.45	19182.25	351.79	148.68	8107.31
0.0	1.0	0.0	2.54	138.50	56.47	3188.93	6.45	19182.25	351.79	143.44	7821.18
0.0	1.0	0.0	2.54	138.50	55.81	3115.20	6.45	19182.25	351.79	141.77	7730.23
0.0	1.0	0.0	2.54	173.50	76.19	5804.98	6.45	30102.25	440.69	193.52	13219.03
0.0	1.0	0.0	2.54	173.50	68.57	4702.04	6.45	30102.25	440.69	174.17	11897.13
0.0	1.0	0.0	2.54	173.50	72.73	5289.25	6.45	30102.25	440.69	184.73	12618.16
0.0	1.0	0.0	3.18	138.50	20.34	413.67	10.08	19182.25	439.74	64.58	2816.95
0.0	1.0	0.0	3.18	138.50	18.90	357.12	10.08	19182.25	439.74	60.00	2617.32
0.0	1.0	0.0	3.18	138.50	19.20	368.64	10.08	19182.25	439.74	60.96	2659.20
0.0	1.0	0.0	3.18	173.50	25.67	658.87	10.08	30102.25	550.86	81.50	4453.47
0.0	1.0	0.0	3.18	173.50	23.65	559.10	10.08	30102.25	550.86	75.07	4102.46
0.0	1.0	0.0	3.18	173.50	25.67	658.87	10.08	30102.25	550.86	81.50	4453.47
0.0	0.0	1.0	2.54	138.50	34.78	1209.83	6.45	19182.25	351.79	88.35	4817.39
0.0	0.0	1.0	2.54	138.50	36.92	1363.31	6.45	19182.25	351.79	93.78	5113.84
0.0	0.0	1.0	2.54	138.50	39.02	1522.90	6.45	19182.25	351.79	99.12	5404.88
0.0	0.0	1.0	3.18	138.50	13.11	172.00	10.08	19182.25	439.74	41.64	1816.39
0.0	0.0	1.0	3.18	138.50	12.09	146.18	10.08	19182.25	439.74	38.39	1674.56
0.0	0.0	1.0	3.18	138.50	13.19	173.89	10.08	19182.25	439.74	41.87	1826.37
0.0	0.0	1.0	3.18	173.50	16.55	273.96	10.08	30102.25	550.86	52.55	2871.72
0.0	0.0	1.0	3.18	173.50	16.22	262.97	10.08	30102.25	550.86	51.49	2813.51
0.0	0.0	1.0	3.18	173.50	16.11	259.45	10.08	30102.25	550.86	51.14	2794.63

1.0	0.0	0.0	2.54	138.50	126.32	15955.67	6.45	19182.25	351.79	320.84	17494.73
1.0	0.0	0.0	2.54	138.50	117.07	13706.12	6.45	19182.25	351.79	297.37	16214.63
1.0	0.0	0.0	2.54	138.50	111.63	12460.78	6.45	19182.25	351.79	283.53	15460.46
1.0	0.0	0.0	2.54	173.50	154.84	23975.03	6.45	30102.25	440.69	393.29	26864.51
1.0	0.0	0.0	2.54	173.50	150.00	22500.00	6.45	30102.25	440.69	381.00	26025.00
1.0	0.0	0.0	2.54	173.50	154.84	23975.01	6.45	30102.25	440.69	393.29	26864.50
1.0	0.0	0.0	3.18	138.50	38.71	1498.44	10.08	19182.25	439.74	122.90	5361.29
1.0	0.0	0.0	3.18	138.50	41.38	1712.24	10.08	19182.25	439.74	131.38	5731.03
1.0	0.0	0.0	3.18	138.50	40.34	1627.00	10.08	19182.25	439.74	128.07	5586.55
1.0	0.0	0.0	3.18	173.50	50.53	2552.91	10.08	30102.25	550.86	160.42	8766.31
1.0	0.0	0.0	3.18	173.50	56.47	3188.93	10.08	30102.25	550.86	179.29	9797.64
1.0	0.0	0.0	3.18	173.50	53.33	2844.44	10.08	30102.25	550.86	169.33	9253.33
0.0	1.0	0.0	1.90	138.50	129.73	16829.79	3.63	19182.25	263.84	247.14	17967.56
0.0	1.0	0.0	1.90	138.50	120.00	14400.00	3.63	19182.25	263.84	228.60	16620.00
0.0	1.0	0.0	1.90	138.50	120.00	14400.00	3.63	19182.25	263.84	228.60	16620.00
0.0	1.0	0.0	1.90	173.50	160.00	25600.00	3.63	30102.25	330.52	304.80	27760.00
0.0	1.0	0.0	1.90	173.50	171.43	29387.74	3.63	30102.25	330.52	326.57	29742.85
0.0	1.0	0.0	1.90	173.50	165.52	27395.95	3.63	30102.25	330.52	315.31	28717.23
0.0	0.0	1.0	1.90	138.50	78.69	6191.87	3.63	19182.25	263.84	149.90	10898.34
0.0	0.0	1.0	1.90	138.50	82.76	6848.99	3.63	19182.25	263.84	157.66	11462.07
0.0	0.0	1.0	1.90	138.50	85.71	7346.95	3.63	19182.25	263.84	163.29	11871.43
0.0	0.0	1.0	1.90	173.50	104.35	10888.46	3.63	30102.25	330.52	198.78	18104.34
0.0	0.0	1.0	1.90	173.50	100.00	10000.00	3.63	30102.25	330.52	190.50	17350.00
0.0	0.0	1.0	1.90	173.50	100.00	10000.00	3.63	30102.25	330.52	190.50	17350.00
0.0	0.0	1.0	2.54	173.50	43.24	1869.98	6.45	30102.25	440.69	109.84	7502.70
0.0	0.0	1.0	2.54	173.50	45.28	2050.55	6.45	30102.25	440.69	115.02	7856.59
0.0	0.0	1.0	2.54	173.50	46.60	2171.74	6.45	30102.25	440.69	118.37	8085.43

DESIGN OF AN INDENTOR VISCOMETER
FOR
DETERMINING THE VISCOSITY OF A FLUID

by

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B.S. Mechanical Engineering, Punjabi University, India, 1964

AN ABSTRACT OF A MASTER'S THESIS

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The mechanics of fluids is dependent upon their fundamental property named as viscosity. The exact information about viscosity is essential in design of pipelines, in injection moulding, extrusion, spraying, bearings etc. Industries which deal with processing of fluids and their by products have often faced difficulties in design of processing system and handling equipment as information available about viscosity of highly viscous fluids is inadequate. Though the commercial viscometers are available for determining viscosity but their ranges are very limited and can not be used for highly viscous fluids. In light of this a viscometer has been designed and fabricated in Kansas State University which is based on the theoratical analysis given by Goudy and Kirmser. The viscometer is designed for testing the viscosity of highly viscous fluids. To varify the satisfactory working of this viscometer the author performed experiments for determining viscosity of fluid samples which were obtained from Brookfield Engineering Laboratories. The experimental data was analysed statistically on computer and results obtained were compared with those obtained by Brookfield Viscometer. The results of two viscometers show large deviation. The apparent discrepancies are mainly attributed to the small size of the containers which directly contradict the assumption made in development of theory based on indentation of semi-infinite fluid surface.

Results of this study indicate that if proper size of container is used the results of two will be quite close. The small containers introduce error due to change of end effects which are mainly due to propagation of the disturbance to the container walls. This could be avoided if highly viscous fluids are used for determining the viscosity. It is hoped that

viscometer designed will give better results even for highly viscous fluids where Brookfield Viscometer can not be used.